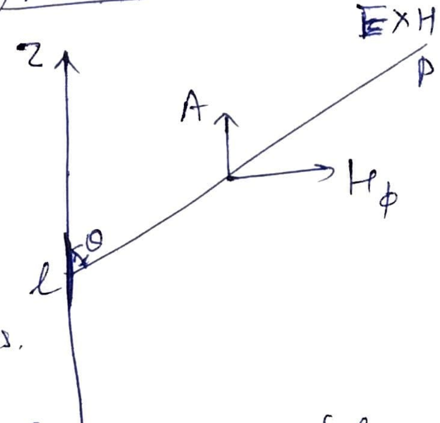
Radiation from an Oscillating Dipole

radiation field produced by an electric oscillating dipole?

This field has many important applications



Many practical radiating systems → may be considered to be made up by putting together a large number of such dipoles.

It contains many ~~[illegible]~~ features useful in quantum theory of emission of radiation by atoms, molecules and nuclei.

Consider two small spheres at the two ends of a short wire. Suppose the charge is transferred periodically from one sphere to the other, the time variations being harmonic, i.e.

$$q = q_0 \exp(i\omega t) \quad \text{--- (14)}$$

$q_0 \rightarrow$ amplitude of oscillating charge.

$\omega \rightarrow$ angular frequency of oscillation

Let us assume that the wavelength of the radiation produced is large compared with the length of the wire l, i.e.

$$\lambda = \frac{2\pi c}{\omega} \gg l \quad \text{or} \quad \frac{2\pi}{\omega} = T \gg \frac{l}{c}$$

This means that the time l/c taken for a signal to propagate along the wire, from one end to the other, is very much less than that over which the source current changes appreciably. We may take the current I to be same at all points along its length.

Now $I = \frac{dq}{dt} = i\omega q_0 \exp(i\omega t)$ ---(15)

The charge oscillating between two spheres is equivalent to an oscillating dipole moment p

$$|\vec{p}| = ql = q_0 l \exp(i\omega t)$$

$$= p_0 \exp(i\omega t) = \frac{Il}{i\omega} \quad \text{---(16)}$$

where $p_0 = q_0 l$.

Let us now determine: (i) how the radiation field of this dipole is distributed throughout space and (ii) the total power radiated

The dipole is shown in figure. The wire lie along the z-axis. The origin of the coordinate system coincides with the centre of the wire.

We need to find the field values at a point P specified by the position vector r.

First we need to find the retarded potentials.

The vector potential A is given by

$$A(r, t') = \frac{\mu_0}{4\pi} \int_V \frac{j\left(r', t - \frac{|r - r'|}{c}\right)}{|r - r'|} d\tau$$

The integration is over the volume occupied by the current, Now

$$j\left(r', t - \frac{|r - r'|}{c}\right) d\tau = I\left(r', t - \frac{|r - r'|}{c}\right) \hat{e}\, dz$$

Since, the current is always along the z-axis, r' may be replaced by z. Hence,

$$A(r, t) = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{I\left(z, t - \frac{|r - \hat{e}_z z|}{c}\right)}{|r - \hat{e}_z z|} e_z\, dz \quad \text{---(17)}$$

Since $r >> l$, we may neglect $\hat{e}_z z$ w.r.t. r and put the denominator in eqn (17) as r and replace the time $t - \frac{|r - \hat{e}_z z|}{c}$ at which the current density in the wire is to be measured by the time $t - r/c$.

Thus

$$A(r,t) = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I(z, t - \frac{r}{c})}{r}\,\hat{e}_z\,dz$$

$$= \frac{\mu_0}{4\pi}\,\hat{e}_z l\,\frac{I(t - r/c)}{r} \quad \text{---(18)}$$

because the current is the same at all points along the wire. This equation shows that the vector potential A is everywhere parallel to l.

Thus the components of the vector potential are known.

$$A_x(r,t) = 0 \;; \quad A_y(r,t) = 0$$

and $$A_z(r,t) = \frac{\mu_0 l}{4\pi r}\, i\omega q_0 \exp\{i\omega(t - \tfrac{r}{c})\}$$

$$= \frac{\mu_0}{4\pi r}\, i\omega p_0 \exp(i\omega t)\exp(-i\omega r/c)$$

$$= \frac{\mu_0}{4\pi r}\, i\omega\, p(t)\, e^{-ikr} = \frac{\mu_0}{4\pi}\,\cancel{p(t)}\,\frac{e^{-ikr}}{r}$$

[here $k = \frac{\omega}{c}$, $p(t) = p_0 \exp(i\omega t)$ and $\dot{p}(t) = \frac{dp(t)}{dt}$]

$$= \frac{\mu_0}{4\pi}\,\dot{p}(t)\,\frac{e^{-ikr}}{r} \quad \text{---(19)}$$

The scalar potential ϕ can easily be obtained from the Lorentz condition

$$\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t} \quad \text{---(20)}$$